

# Reachability Analysis of Closed-Loop Switching Power Converters

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December 16, 2012

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Background

Modeling

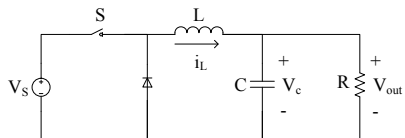
Analysis

# Switching Power Converters

1. DC-DC Converters
  - ▶ Buck converter
  - ▶ Boost converter
  - ▶ Buck-Boost converter
2. Rectifiers (AC-DC)
3. Inverters (DC-AC)
4. Transformers

# Buck Converter

## Circuit Diagram

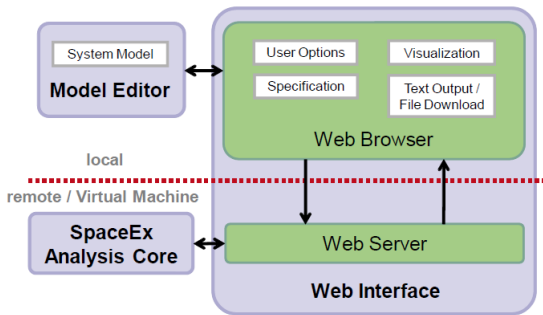


# Design Verification of Switching Power Converters

1. Traditional simulation methods
  - ▶ Simulink/Stateflow
  - ▶ PSpice
  - ▶ Monte Carlo
2. Hybrid system verification tools
  - ▶ SpaceEx

# SpaceEx

## Software Architecture



# SpaceEx Reachability Algorithm

- ▶ Symbolic States
- ▶ Control:
  1. Maximum iterations
  2. Relative and absolute error

# SpaceEx Interface

## Setting Maximum Iterations and Relative and Absolute Errors

Model	Specification	Options	Output	Advanced
Scenario	LGG Support Function			?
Directions	<input type="radio"/> box <input checked="" type="radio"/> oct <input type="radio"/> uni			?
Clustering percentage	100			?
Aggregate sets	Convex hull			?
Sampling time	0.00001			?
Flowpipe tolerance	0.001			?
Initial samples	50			?
Local time horizon	1			?
Max. iterations	15			?

Model	Specification	Options	Output	Advanced
Relative error	1.0e-12			?
Absolute error	1.0e-15			?
ODE tolerance rel.				
ODE tolerance abs.				
Additional options				?
SpaceEx Version	SpaceEx State Space Explorer, v0.9.7beta, compiled Nov 26 2012, 13:16:57, 64-bit float, 64-bit precise float			
Interface Version	SpaceEx Web Interface v1.0-BETA1.4 // 2011-10-24			
3D Display library	<a href="http://www.javaview.de">www.javaview.de</a>			



# SpaceX Interface

## Initial States and Output Specification

Model Specification Options Output Advanced

System buck Update ?

**buck**

- Controlled : *il*, *vc*, *t*, *gt*
- Constant-Dynamics : *vs*, *vref*, *delta*, *tmax*
- Base-components : *buck\_template\_1*

Initial states ?

```
loc(buck_template_1)==charging & il == 0 & vc ==0 & t == 0 & gt
== 0 & vs==12 & tmax ==0.001 & vref==5 & delta==0.1
```

Forbidden states ?

Model Specification Options Output Advanced

Output format 2D (gen) ?

Output variables *il*, *vc*, *gt* ?

Output error  ?

Generate PDF file

Echo the generated command lines to the console

Verbosity Debug 4 ?

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## Model of Open-Loop Configuration

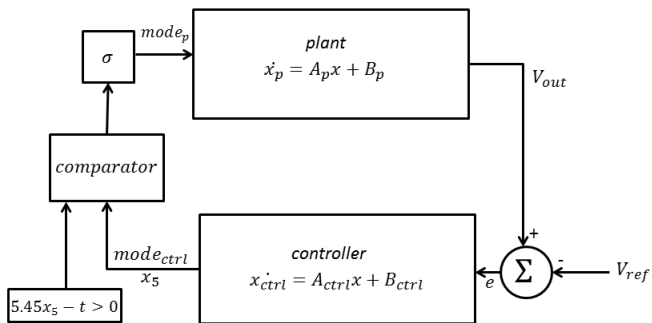
$$x = \begin{bmatrix} i_L \\ V_c \end{bmatrix}$$

$$A_o = A_c = \begin{bmatrix} 0 & -\frac{1}{L} \\ \frac{1}{C} & -\frac{1}{RC} \end{bmatrix}$$

$$B_c = \begin{bmatrix} 1/L \\ 0 \end{bmatrix} V_s \text{ or } B_o = \begin{bmatrix} 0 \\ 0 \end{bmatrix} V_s$$

# Linear Controller

## Block Diagram



# Linear Controller

## System Model

$$A_{ctrl} = \begin{bmatrix} -\frac{1}{p_1} & 0 & 0 \\ -\frac{p_2}{p_1 p_3} + \frac{1}{p_3} & -\frac{1}{p_3} & 0 \\ -\frac{p_2 p_4}{p_1 p_3 p_5} + \frac{p_4}{p_3 p_5} & \frac{-p_4}{p_3 p_5} + \frac{1}{p_5} & 0 \end{bmatrix}$$

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \quad B_{ctrl} = \begin{bmatrix} \frac{1}{p_1} \\ \frac{p_2}{p_1 p_3} \\ \frac{p_4 p_2}{p_1 p_3 p_5} \end{bmatrix}$$

# Linear Controller

## Composed System

$$\dot{x} = A_C \cdot x_C + B_{comp} (V_{ref} - V_{out})$$

# Linear Controller

## Composed System

$$A_c = \begin{bmatrix} 0 & \frac{-1}{L} & 0 & 0 & 0 & 0 \\ \frac{1}{C} & \frac{-1}{RC} & 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{1}{p_1} & -\frac{1}{p_1} & 0 & 0 \\ 0 & 0 & -\frac{p_2}{p_1 p_3} & -\frac{p_2}{p_1 p_3} + \frac{1}{p_3} & -\frac{1}{p_3} & 0 \\ 0 & 0 & -\frac{p_4 p_2}{p_1 p_3 p_5} & -\frac{p_2 p_4}{p_1 p_3 p_5} + \frac{p_4}{p_3 p_5} & -\frac{p_4}{p_3 p_5} + \frac{1}{p_5} & 0 \end{bmatrix}$$

$$\dot{x}_c = \begin{bmatrix} \dot{i}_L \\ \dot{V}_c \\ x_1 \\ x_2 \\ x_3 \end{bmatrix} \quad B_c = \begin{bmatrix} \frac{V_s}{L} \\ 0 \\ \frac{1}{p_1} \cdot V_{ref} \\ \frac{p_2}{p_1 p_3} \cdot V_{ref} \\ \frac{p_4 p_2}{p_1 p_3 p_5} \cdot V_{ref} \end{bmatrix} \quad B_o = \begin{bmatrix} 0 \\ 0 \\ \frac{1}{p_1} \cdot V_{ref} \\ \frac{p_2}{p_1 p_3} \cdot V_{ref} \\ \frac{p_4 p_2}{p_1 p_3 p_5} \cdot V_{ref} \end{bmatrix}$$

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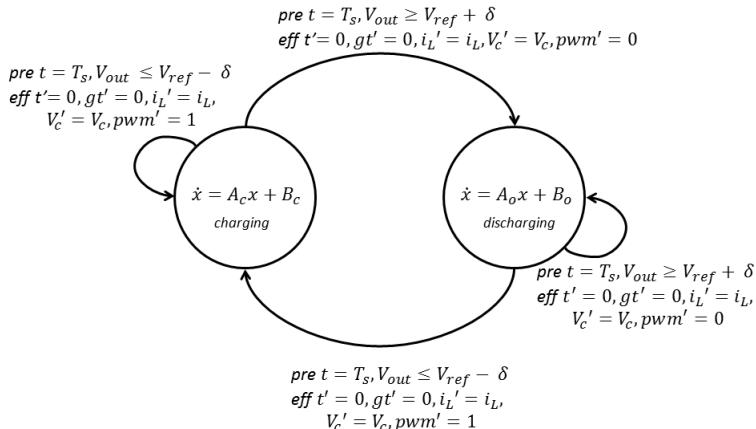
Modeling

Analysis



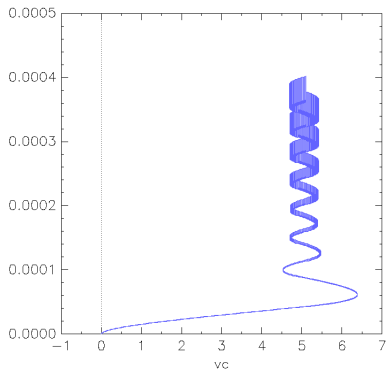
# Hysteresis Controller

## Hybrid Model of Buck Converter and Hysteresis Controller



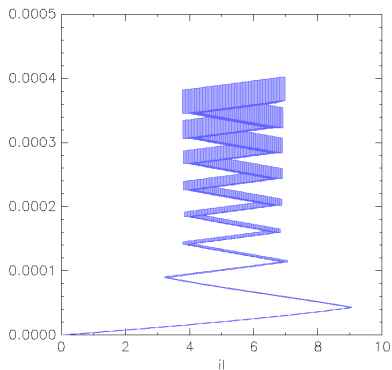
# Results of Hysteresis Controller

Capacitor Voltage vs. Global Time



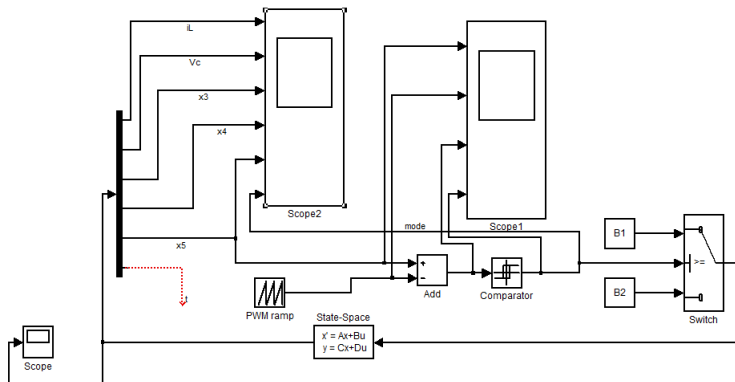
# Results of Hysteresis Controller

Inductor Current vs. Global Time



# Simulink Results for Composed System

## Simulink Model



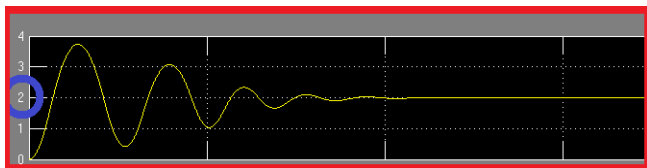
# Simulink Results for Composed System

## Simulink Output



# Simulink Results for Composed System

Capacitor Voltage vs. Global Time



## Conclusion and Future Work

- ▶ Open-loop and hysteresis controller configurations
- ▶ Modeling flexibility
- ▶ Overapproximation issues

# References

[1] [2] [3]



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► Questions?