# Synchronized Behavior in Networks of Hybrid Systems

Sean Shahkarami

December 16, 2012

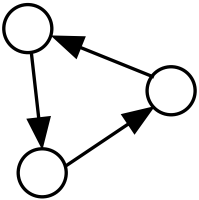
# Informal Motivational Question

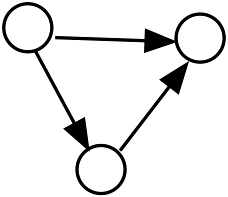
What can the configuration of a network of hybrid systems tell us about synchronous behavior amoung its nodes?

# Simple Example

## Example

Are there general conditions ensuring synchronous behavior of the nodes in either of these networks of hybrid systems?





# Project Foundations

Our first step towards an answer is built on the following two ingredients

- Graph Fibrations
- Hybrid Networks

# Graph Fibrations

## Definition

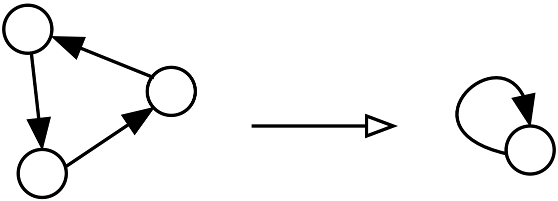
A *graph fibration* is a graph homomorphism $\phi : G \to G'$ with the property that for all $v \in G$ and all edge $e' \in G'$ with target $\phi(v)$ there exists a *unique* edge $e \in G$ such that $\phi(e) = e'$.

# Graph Fibrations

## Definition

A *graph fibration* is a graph homomorphism $\phi : G \to G'$ with the property that for all $v \in G$ and all edge $e' \in G'$ with target $\phi(v)$ there exists a *unique* edge $e \in G$ such that $\phi(e) = e'$.

## Example



# Main Intuition

The most important idea to keep in mind with graph fibrations is the following motto

$$\textit{Proper Graph Fibrations} \implies \textit{Synchronous Behaviors}$$

# Useful Criteria for Graph Fibrations

**Theorem**

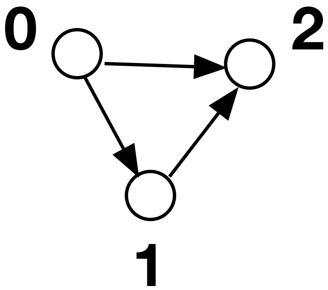
*If $\phi : G \to G'$ is a graph fibration then for each vertex $v \in G$ there is a bijection between the incoming edges of $v$ and the incoming edges of $\phi(v)$.*

In other words, *the local incoming edge structure of all the verticies above a base vertex is the same.*

# Simple Counterexample

## Example

Our second example admits no proper graph fibration by a simple counting argument and the above criteria.



# Hybrid Networks

## Definition

A *hybrid network* consists of

- A directed graph $G$.
- An assignment $\mathcal{A}(v)$ of hybrid automata to each vertex $v \in G$ with the modification that the continuous dynamics of $\mathcal{A}(v)$ is allowed to depend on the state of $\mathcal{A}(w)$ for each incoming neighbor of $v$.

# Associated Hybrid Automata

To a hybrid network $(G, \mathcal{A}(v))$ we associate an ordinary hybrid automata $G_{||}$ defined by

- The variables of $G_{||}$ is the disjoint union of the variables of each $\mathcal{A}(v)$.
- The state space is the product of the state spaces of the $\mathcal{A}(v)$.
- The start states are the product of the start states of the $\mathcal{A}(v)$.
- The actions are the union of the actions of the $\mathcal{A}(v)$.
- A transition $x \xrightarrow{a} y$ exists if and only if for each $v \in G$ either (1) $a \in A_v$ and $x\lceil X_v \xrightarrow{a} y\lceil X_v$ or (2) $a \notin A_v$ and $x\lceil X_v = y\lceil X_v$.
- Trajectories are solutions to the differential equations (now without parameters).

# Main Result

**Theorem**

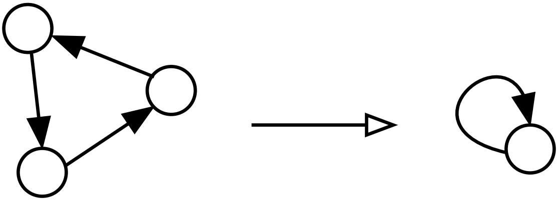
*Given hybrid networks $(G, \mathcal{A}_v)$ and $(G', \mathcal{A}'_v)$ and a surjective graph fibration $\phi : G \to G'$ such that $\mathcal{A}_v = \mathcal{A}_{\phi(v)}$, there exists a forward simulation from $G'_{||}$ to $G_{||}$.*

# Main Result

### Theorem

*Given hybrid networks $(G, \mathcal{A}_v)$ and $(G', \mathcal{A}'_v)$ and a surjective graph fibration $\phi : G \to G'$ such that $\mathcal{A}_v = \mathcal{A}_{\phi(v)}$, there exists a forward simulation from $G'_{||}$ to $G_{||}$.*

### Example



# Closing

Formal result aside, the main point to take away from this project is motto

$$\textit{Proper Graph Fibrations} \implies \textit{Synchronous Behaviors}$$