ECE/CS 584: Verification of Embedded Computing Systems

Lecture 02

Sayan Mitra

Propositional Logic Summary

- **Syntax** (rules for constructing well formed sentences)
  - Countable set of (atomic) propositions PS: P1, P2, P3, …
  - S = $\mathrm{True} \mid p_1 \mid \neg S_1 \mid S_1 \wedge S_2$ | (S_1)
- **Semantics** defines a truth value functions or **valuations** v that maps each proposition PS to a truth value (T or F), v: PS→ {T, F} and by extension a valuation v':PROPS→{T,F}
- A proposition A is **valid** v'(A) = T for all valuations v. A is also called a **tautology**
- A proposition is **satisfiable** if there is a valuation (or truth assignment) v such that v(A) = T.
- Checking (un)satisfiability is called **boolean satisfiability problem** (SAT).
- SAT is (decidable) **NP-complete** problem

Predicate Logic or First Order Logic

- Syntax defined by a signature of **predicate** & **function** symbols
  - Variables
  - Predicate symbols with some **valence** or **arity**
    - a is predicate of 0-arity, like propositions
    - P(x) is a predicate of 1-arity
    - Q(x,y) is a predicate of 2-arity
  - Function symbols of some **valence**,
    - Function symbols of 0 arity are called constants
    - f(x) is a function of arity 1, e.g., -x
  - A **term** t ::= x | f(t1,t2,t3,…), where t1, t2, t3, … are terms
  - A **formula** φ ::= a | P(x) | Q(x,y) | t1 = t2 |$\neg\varphi$ | (φ1 ⇒ φ2) | … | … | $\forall x \varphi$ | $\exists x \varphi$
- Example of **Well Formed Formula**
  - $\exists x\ P(x)$, ∀x∀y(E(x, y) ⇒ E(y, x)), $\forall x\ y\ Q\big(x, f(y)\big) \equiv Q(f(y), x)$
- **Bounded** and **unbounded variables, closed formulas**

Semantics

- An **interpretation or a model M** of a FOL formula assigns meaning to all the non-logical symbols and a domain for the variables (i.e., the variables, the predicate symbols, and the function symbols)
  - D: Domain of discourse
  - For each variable x, a valuation v(x) gives a value in D
  - Each function symbol f of arity n is assigned a function $D^n \rightarrow D$
  - Each predicate symbol P of atity n is assigned a predicate $D^n \rightarrow \{T, F\}$
- If formula φ evaluates to T with model M, then we say M **satisfies** φ, $M \vDash \varphi$ and φ is said to be **satisfiable**
- φ is **valid** if it is true for every interpretation

Example (Un)Decidable Classes

	Prefix	# of n-ary predicate symbols	# of n-ary function symbols	With Equality	Name
Undecidable	$\forall\exists\forall$	ω, 1	0	N	Kahr 1962
	$\forall^3\exists$	ω, 1	0	N	Suranyi 1959
	$\forall^*\exists$	0,1	0	N	Kalmar-Suranyi 1950
	$\forall\exists\forall\exists^*$	0,1	0	N	Gurevich 1966
	$\forall$	0	2	Y	Gurevich 1976
	$\forall$	0	0, 1	Y	Gurevich 1976
	$\forall^2\exists$	ω, 1	0	Y	Goldfarb 1984
Decidable	$\exists^*\forall^*$	all	0	Y	Ramsey 1930
	$\exists^*\forall\exists^*$	all	all	N	Maslov-Orevkov 1972
	$\exists^*$	all	all	Y	Gurevich 1976
	all	ω	ω	N	Lob 1967

Theory of Time Input/Output Automata

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Roadmap

- Syntax
- Semantics
- Abstraction, Implementation
- Simulations
- Composition
- Substitutivity

Variables and Valuations

- A variable *x* is a name for a state component
- *type(x)*
- A set of variables X
- A valuation for X maps each x ∈ X to an element in type(x)
- $val(X)$: set of all valuations of X

- x:$\mathbb{R}$
- color:{R,G,B}
- clock:$\mathbb{R}^{\geq 0}$
- X = {x,color,clock}
- **x** = <x → 5.5, color →G, clock → 12>
- **y** = <x → 7.90, color →G, clock → 1>
- **x.**color = G, **x.**x = 5.5, **y.**x = 7.90

Trajectories

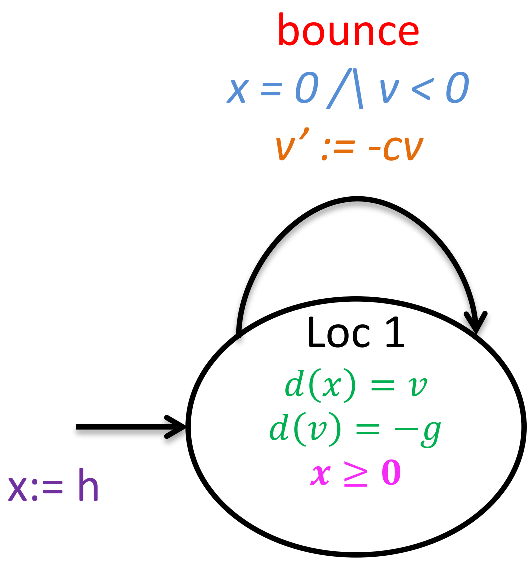
- Time = $\mathbb{R}^{\geq 0}$
- Time interval = [a,b]
- A trajectory for X is a function τ: $[0, t] \rightarrow val(X)$, where [0,t] is an interval
- $\tau.dom = [0, t]$
- x is **continuous (or analog)** if all its trajectories are piecewise continuous
- **Discrete** if they are piecewise constant
- Notations: τ.fstate, τ.lstate, τ.x, τ.X
- Prefix, suffix, concatenation

Hybrid Automata (a.k.a Timed Automata Kaynar, et al. 2005)

$\boldsymbol{\mathcal{A}} = (X, Q, \Theta, E, H, \mathcal{D}, \mathcal{T})$

- X**:** set of internal variables
- $Q \subseteq val(X)$ set of states
- $\Theta \subseteq Q$ set of start states
- *E,H* sets of internal and external actions, A= E ∪ H
- $\mathcal{D} \subseteq Q \times A \times Q$
- $\mathcal{T}$*:* set of trajectories for X which is closed under prefix, suffix, and concatenation

Bouncing Ball



Automaton Bouncingball(c,h,g)

variables: analog x: Reals := h, v: Reals := 0

states: True

actions: external bounce

transitions:

bounce

pre *x = 0 /\ v < 0*

eff v := -cv

trajectories:

evolve d(x) = v; d(v) = -g

invariant $x \geq 0$

Graphical Representation used in many articles

TIOA Specification Language (close to PHAVer & UPPAAL's language)

Trajectory Semantics