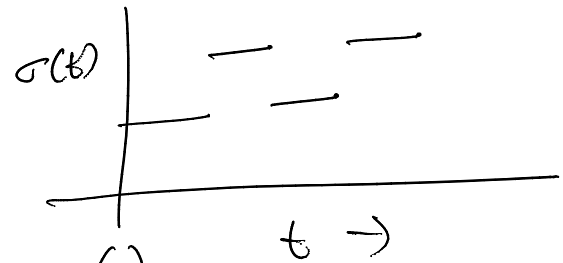
Thursday, November 01, 2012
11:02 AM

Plan

- Average Dwell time theorem for AS of switched systems

$\dot{x} = f_i(x) \quad i \in \mathcal{I}$

$\sigma : [0, \infty) \to \mathcal{I}$



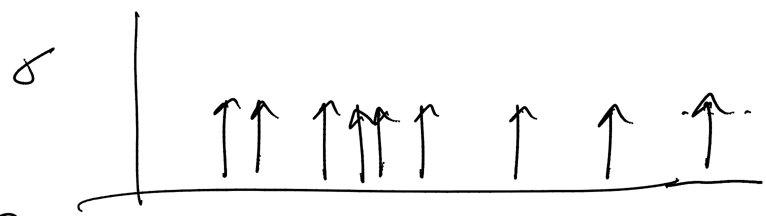
switched ~~t~~ system $\quad \dot{x} = f_{\sigma(t)}(x)$ —— (1)

Def Average Dwell Time (ADT)

σ has ADT $\tau_a > 0$ iff

$\exists N_0 \in \mathbb{N}$ s.t $\forall t, T > 0 \quad N_\sigma(t, T) \le N_0 + \dfrac{T-t}{\tau_a}$

\# Extra switches



Thm (1) $\forall i \in \mathcal{I} \quad \exists V_i : \mathbb{R}^n \to \mathbb{R}$

s.t $\quad \dot{V}_i \le -2\lambda_0 V_i$

$$\boxed{\frac{\partial V_i}{\partial x} f_i(x)} \le -2\lambda_0 V_i(x(t))$$

$\dot{V} \le 0$

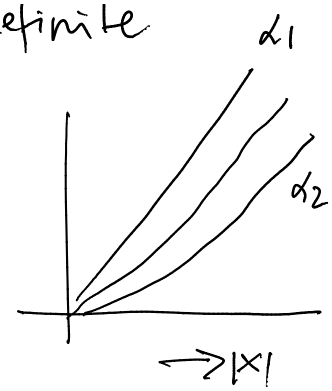
< 0

(2) $\exists \mu > 0$ s.t $\forall i, j \in \mathcal{I} \ \forall x \in \mathbb{R}^2$

$$V_i(x) \le \underline{\mu} V_j(x)$$

(3) $\exists \alpha_1 \alpha_2 : \mathbb{R} \to \mathbb{R}$ monotonic, positive definite $\quad \alpha_1$

(3) $\exists\ \alpha_1\, \alpha_2 : \mathbb{R} \to \mathbb{R}$ monotonic, positive definite



$\forall i \qquad \underline{\alpha_1(|x|) \leq V_i(x) \leq \alpha_2(|x|)}$

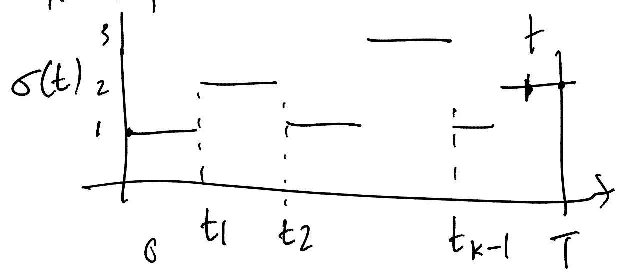
(4) σ has ADT $\tau_a \geq \dfrac{\log \mu}{2\lambda_0}$

Then (1) is GAS.

<u>Proof Sketch</u> fix a switching signal σ satisfies (4).. fix time T

Sayan Mitras at 11/1/2012 11:19 AM

Switching times $t_1\ t_2 \cdots t_{N_\sigma(0,T)}$



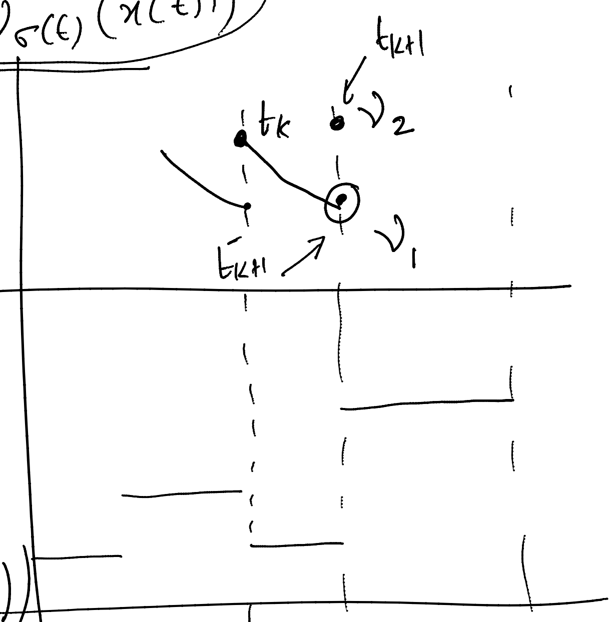
Define aux fn

$$W(t) = e^{2\lambda_0 t}\, V_{\sigma(t)}(x(t))$$ ← piecewise differentiable

$$\dot{W} = 2\lambda_0 e^{2\lambda_0 t} V_{\sigma(t)}(x(t)) + e^{2\lambda_0 t}\, \dot{V}_{\sigma(t)}(x(t))$$ [between switching times

$$\leq \underline{2\lambda_0 e^{2\lambda_0 t} V_{\sigma(t)}(x(t)) - 2\lambda_0 e^{2\lambda_0 t} V_{\sigma(t)}(x(t))}$$

$$\leq 0$$



W is non increasing between switching times

$$W(t_{k+1}) = e^{2\lambda_0 t_{k+1}}\, V_{\sigma(t_{k+1})}(x(t_{k+1}))$$

$$W(t_{k+1}) \leq \mu e^{2\lambda_0 t_{k+1}}\, V_{\sigma(t^-_{k+1})}(x(t_{k+1}))$$ — by (2) t_{k+1}

$$= \mu e^{2\lambda_0 t^-_{k+1}}\, V_{\ldots}(x(t^-_{k+1}))$$

$$\sigma(t_{k+1}^-)$$

$$= \mu W(t_{k+1}^-)$$

$$\leq \mu W(t_k)$$ as W is non increasing between switches

$$\underline{W(T^-)} \leq \mu^{N_\sigma(0,T)} W(0)$$ by induction

$$e^{2\lambda_0 T} V_{\sigma(T^-)}(x(T)) \leq \mu^{N_\sigma(T,0)} V_{\sigma(0)}(x(0))$$

$$V_{\sigma(T^-)}(x(T)) \leq e^{-2\lambda_0 T + N_\sigma(T,0)\log\mu} V_{\sigma(0)}(x(0))$$

$$\boxed{N_\sigma(0,T) \leq N_0 + \frac{T}{\tau_0} \quad \text{Def of ADT}}$$

$$\leq e^{-2\lambda_0 T + \log\mu\,(N_0 + \frac{T}{\tau_a})} V_{\sigma(0)}(x(0))$$

$$\leq e^{-2\lambda_0 T + \log\mu \cdot \frac{T}{\tau_a}} \underbrace{\mu^{N_0} V_{\sigma(0)}(x(0))}_{C}$$

$$\leq e^{T\left(\frac{\log\mu}{\tau_a} - 2\lambda_0\right)} C$$

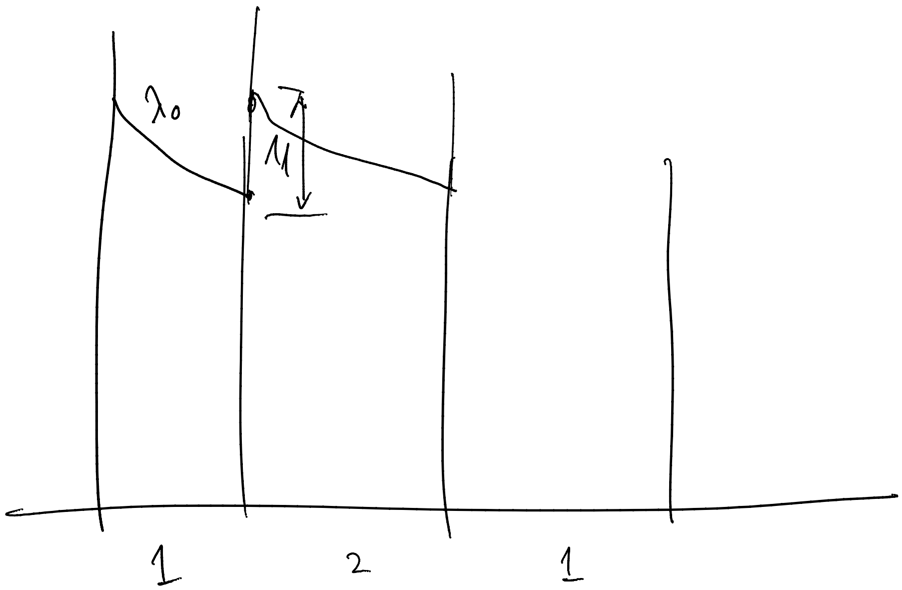
if $\tau_a \geq \frac{\log\mu}{2\lambda_0}$ the exponent is -ve

as $T \to \infty$ $V_{\sigma(T^-)}(x(T)) \to 0$

$V \leq e^{-\lambda T} \mu^{N_0} V_{\sigma(0)}(x(0))$ for some $\lambda \in (0,1)$

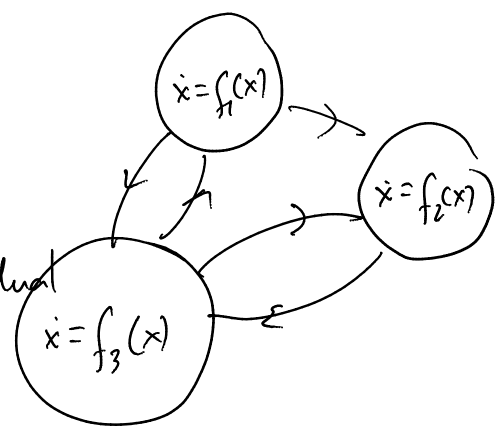
$$V_{\sigma(T^-)} \le e^{-\lambda T} \mu^{N_0} V_{\sigma(0)}(x(0)) \quad \text{for some } \lambda \in (0,1)$$

$$|x(T)| \le \alpha_1^{-1}\left(\mu^{N_0} e^{-\lambda T} \alpha_2(x(0))\right) \quad \blacksquare$$



Applications

- Stability analysis
  - (i) show stability of individual modes
  - (ii) Show system has ADT or satisfies MLF

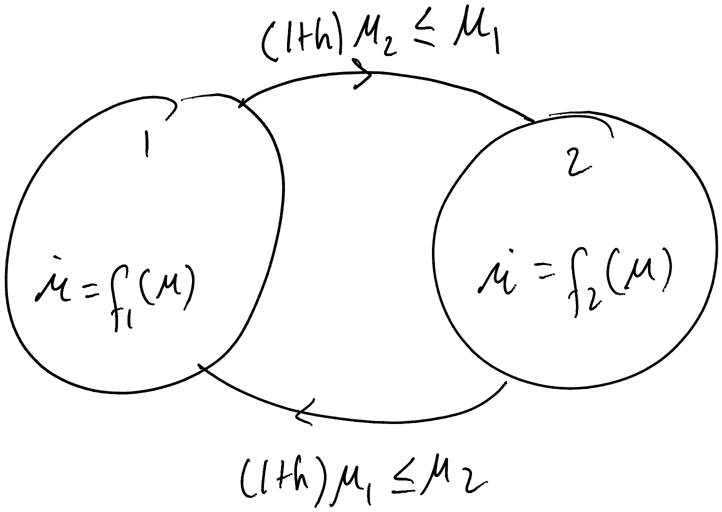


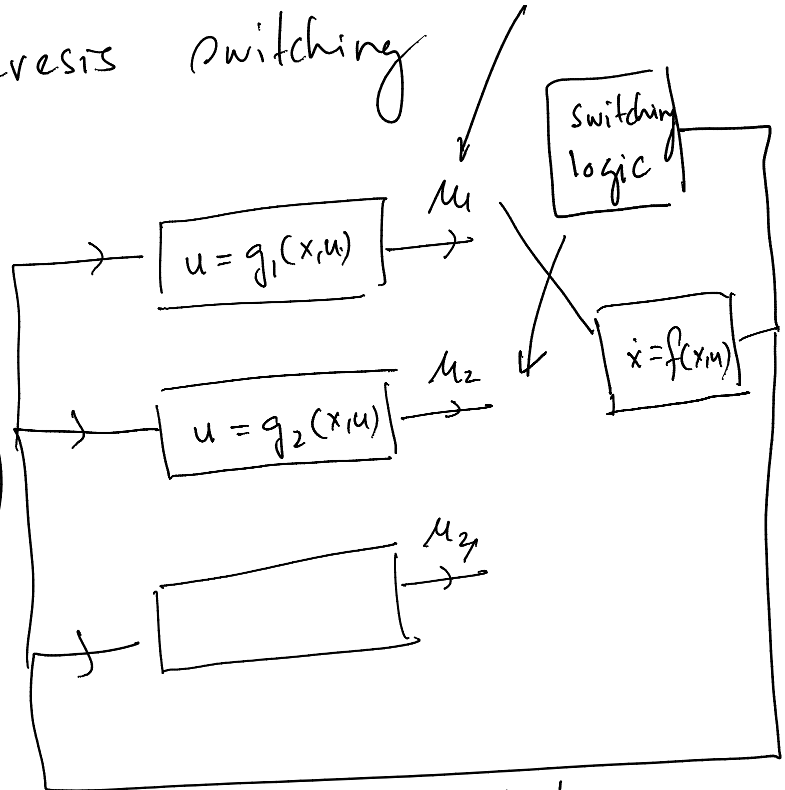
- Designing stable system

Estimates

Scale independent hysteresis switching /

Scale independenden hysteresis switching





Adaptive Control

$\dot{\mu}_1 = a\mu_1$ $\quad\quad$ $\dot{\mu}_2 = a\mu_2$

$\dot{\mu}_2 = 0$ $\quad\quad$ $\dot{\mu}_1 = 0$

---

individual modes are stable