Autonomy in Satellite Systems

- Number of satellites currently in orbit: 3,000
- Speed of a satellite in Geostationary Orbit ~ 11,000 Kmph
- Radio delay ~ minutes (7mins of terror)
- Cost of putting a light satellite in space: Few million dollars
- Software controlled autonomous & complex operations

Image courtesy NASA

Satellite Rendezvous & Collision Avoidance: A Case Study in Verification of Nonlinear Hybrid Systems

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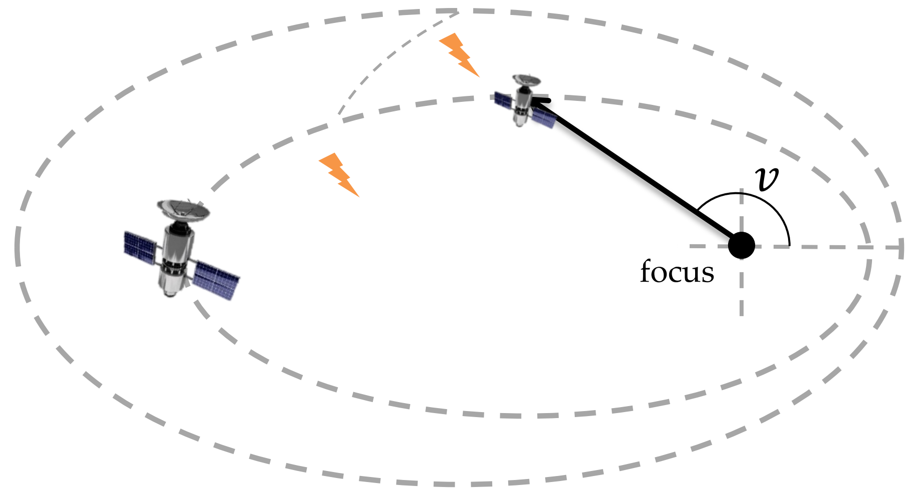
Astrodynamics 101

- A satellite moves in an elliptical 2D orbit in a 3D space
- Given the mass of the Earth, satellite, and the position and velocity of the satellite the orbit is **uniquely** defined
- **Fixing an orbit *o*, the angular speed (*v*) is given by**:
- **Orbital transfers:** Add energy (fire thrusters) to change velocity vector, and therefore, the orbit



$$\dot{v} = f(v,p,e) = \sqrt{\frac{\mu}{p^3}(1 + e\ cos v)^2}$$

$p = a(1 - e^2)$ a: semimajor axis, e: eccentricity

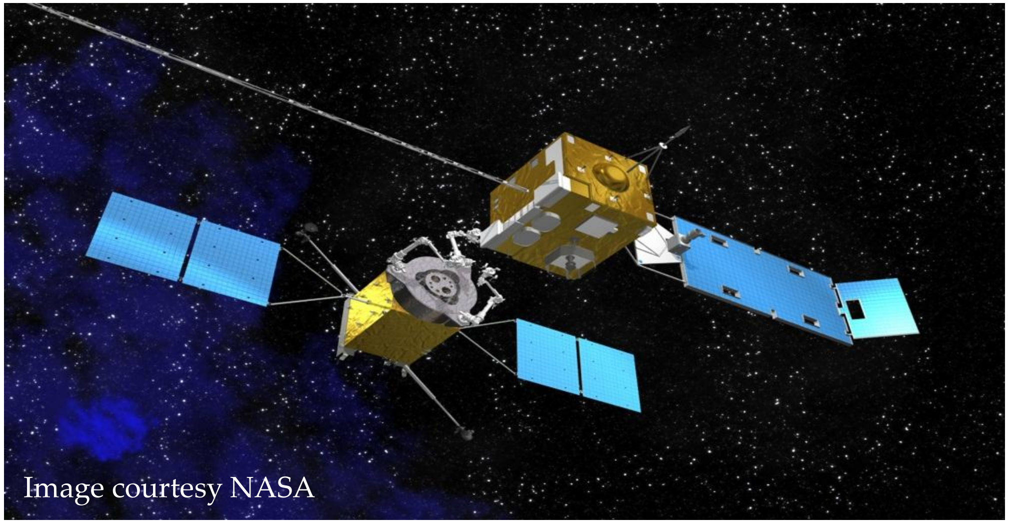
Given *v*, *o* the cartesean coordinates of the satellite are given by *cart(o,v) = (x,y)*

$$r = \frac{p}{1+e\cos v} \qquad x = r\cos v \qquad y = r\sin v$$

Rendezvous and Collision Avoidance

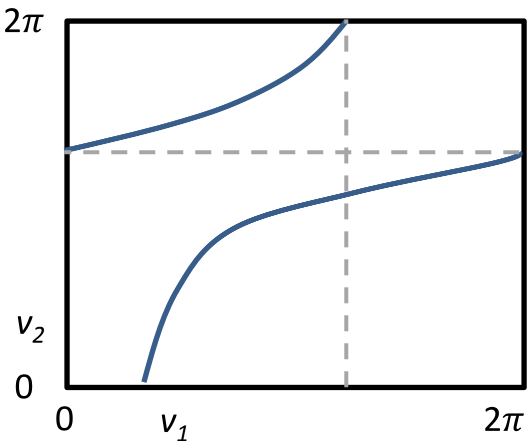
Collision avoidance: Two satellites should never be within d distance

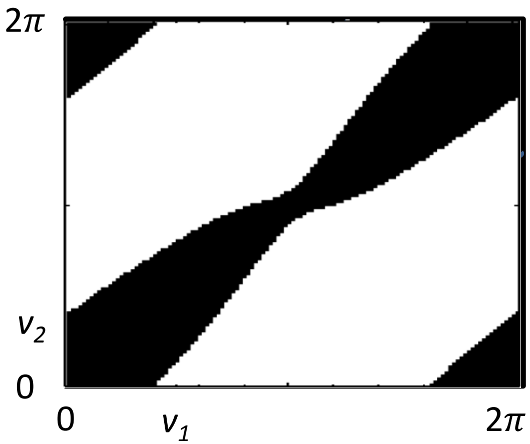
Rendezvous: Two satellites should remain within distance d' over an interval T



Quotient system: 2D rectangle with "wrap-around" transitions

Distance properties mapped to **predicates** on angular positions (for given orbit pair)





Collision avoidance: Unbounded safety, Rendezvous: Bounded reachability

Distance predicate Dist(d, o_1, o_2) = {(v1,v2) | $\|cart(v_1, o_1), cart(v_2, o_2)\| \leq d$ }

A Direct Approach based on Hybrid Model Checking

- Model each satellite in the quotient space as a nonlinear hybrid automaton
  - Trajectories model
  - Transitions model "wrap-around"s and jumps to new orbits

$$\dot{v} = \sqrt{\frac{\mu}{p^3}}(1 + e\,cos v)^2 \;\; with\; v \;\in [0,2\pi]$$

- System *S*: Composition of automata pair
- Compute / approximate the reach set *Reach(S)* Using existing reachability tools like **HyTech, Phaver, SpaceEx**
- Prove collision avoidance by checking emptiness:

  $Reach(S) \cap Dist(d) = \emptyset$

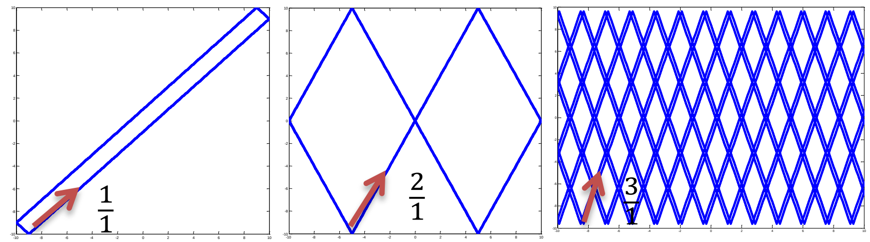
- Prove rendezvous by checking containment:

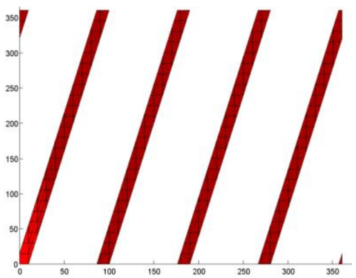
  $Reach(S, t_1, t_2) \subseteq Dist(d')$

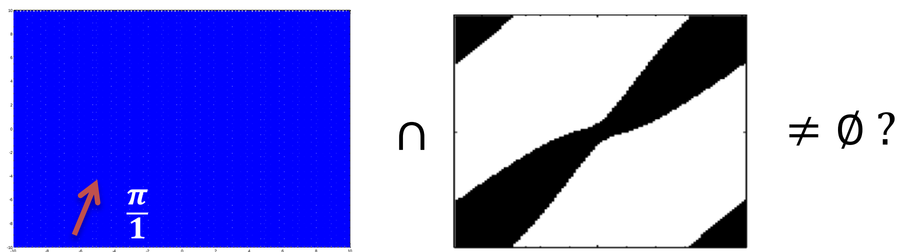
Reach(S), Irrational-ity, and Billiards

What is the reach set of a billiard ball in a unit-square, frictionless table with initial slope s?

If s is irrational then it is **dense** in $[0,1]^2$







For two satellites with circular orbits (e = 0) and periods T_1 and T_2, if T_1/T_2 is

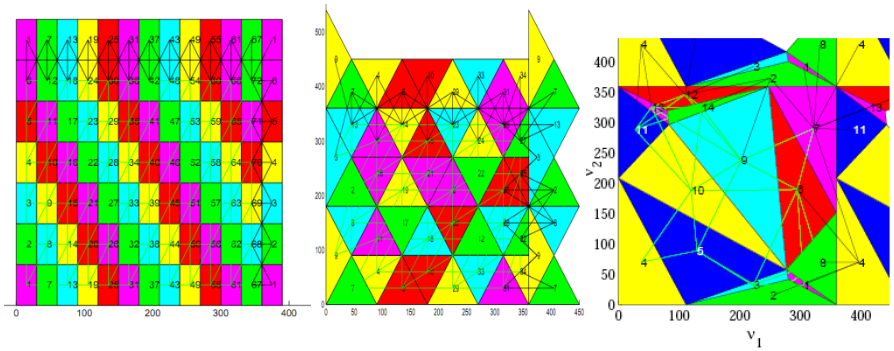
- Rational then OK
- Irrational (incommensurate) then effectively every configuration in (v_1, v_2) is reached

Even for a pair of commensurate elliptical orbits, approximations introduces incommensurate orbits

Compute bounded *Reach(S,t)*

Computing *Reach(S,t)*: Hybridization

- Tools for computing (bounded) Reach(S,t) for nonlinear systems are limited compared to those available for rectangular and linear systems



- Hybridization **[Dang, et al. 07-11]:** A technique for abstracting nonlinear system S with linear/rectangular hybrid system $\bar{S}$

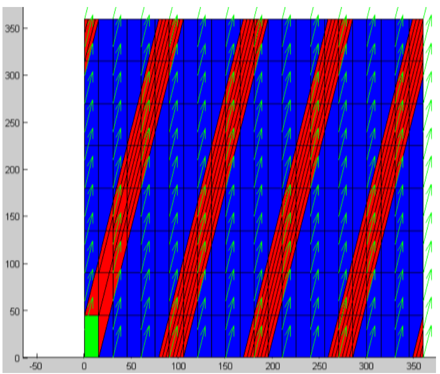
- Key idea:
  1. Partition state space
  2. Over-approximate the nonlinear dynamics within each region with linar/rectangular dynamics
  3. Define appropriate transitions

For a region R define dynamics in R as
Nonlinear dynamics: $\dot{v} = f(v)$
Rectangular $\dot{v} \in [a, b]$
where $a = \min_{v \in R} f(v)$ and $b = \max_{v \in R} f(v)$

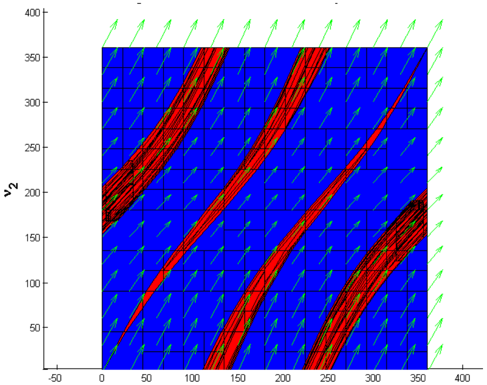
Affine $\dot{v} \in h(v) + U$

$Reach(S,t) \subseteq Reach(\bar{S}, t)$

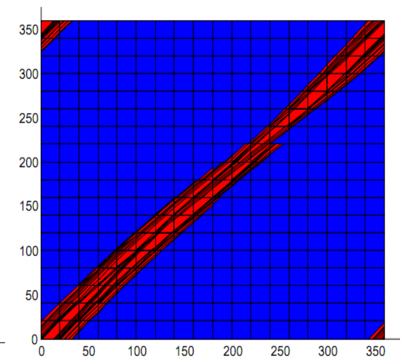
Collision Avoidance Verification



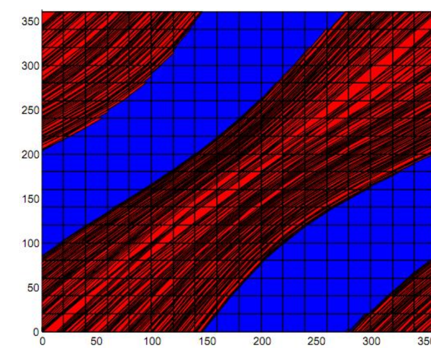
Circular Orbit: $p_1=2$, $p_2=1$



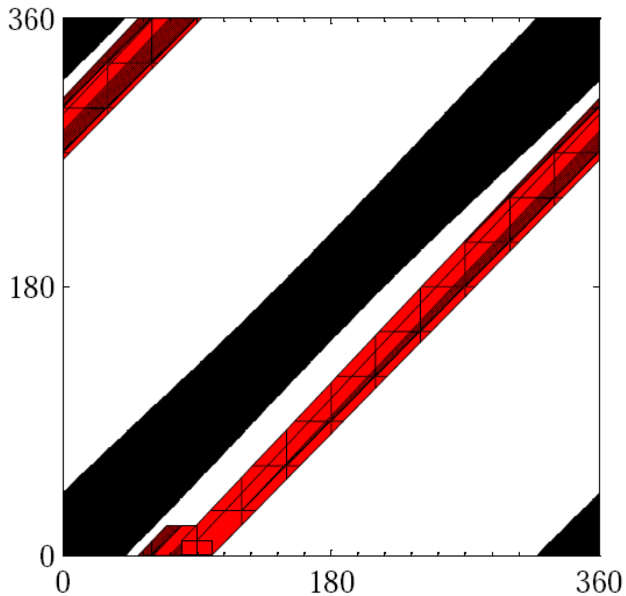
e1=0, e2=0.67



e1=0.1,e2=0.15, 50 iterations



e1=0.1,e2=0.15, 200 iterations

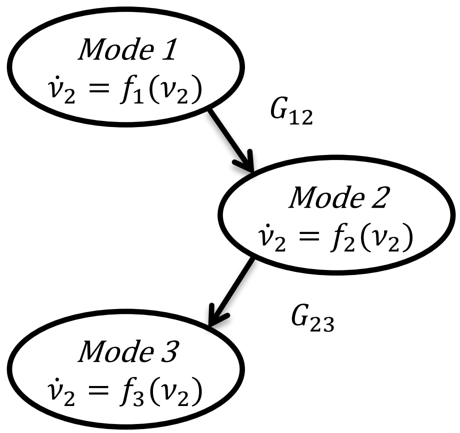


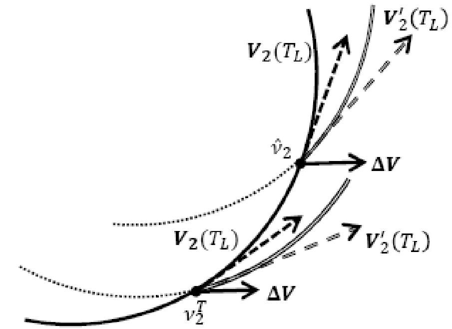
Dist(d): $||\text{cart}(v_1, o_1), \text{cart}(v_2, o_2)|| \leq d$

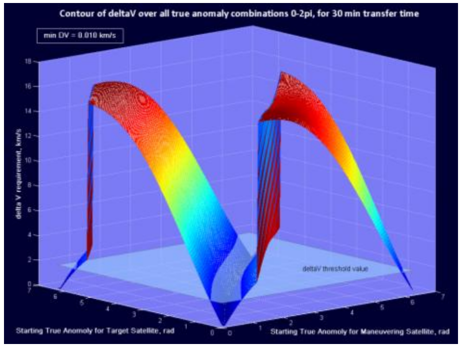
Reach(S,t): **Overapproximation of bounded reach set**

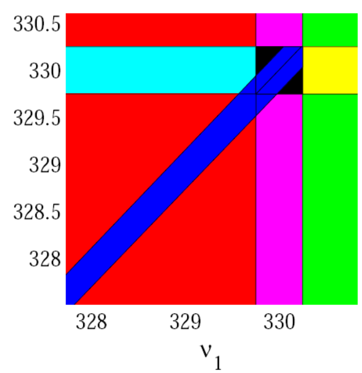
Orbital Transfers

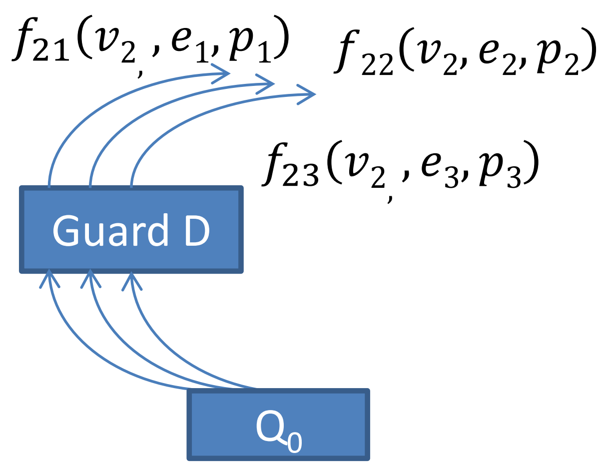
- $G_{12} \subseteq [0,2\pi]$: range of angular positions that minimize energy (Δv) for transfer
- G_{12} computed by solving Lambert's equations
- Additional overapproximation: each point in the guard corresponds to a slightly different orbit
- Mode 2 has to capture the dynamics of this continuum of slightly different orbits for the active satellite
  - The distance predicate for each orbit is also slightly different
- Aggregate continuum of orbits!





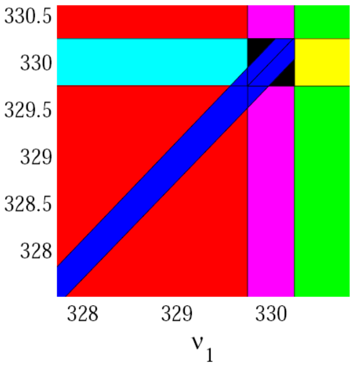






Aggregate orbital dynamics

- Let c^* be the ideal value of v_2 (least energy) Lambert burn point and the corresponding vector Δc^*



- Let L = [c*−ε, c*+ε] be the guard set for the active satellite
- For each $c \in L$ compute the orbital parameters using Δc^*
  - $v(c) = c + \Delta c^*, h(c) = v(c) \times r, e(c) = \frac{1}{\mu}(v(c) \times h(c)) - r(c)/r$
  - $a(c) = \frac{h(c)}{\mu}(1 - e(c)^2), p(c) = a(c)(1 - e(c)^2)$
- All orbital parameters are defined in terms of c
  - $\dot{v} = (1 + f(v, p(c), e(c))$
  - $\dot{v} \in [a, b]$ $where$ $a = \min_{v \in R,\ c \in L} f(v, p(c), e(c))$
  - Exploit periodicity to minimize the function
- Distance predicate

Distance predicate *Dist(d, o₁)* = {(v₁,v₂) | For all o₂ ||cart(v₁, o₁), cart(v₂, o₂)|| ≤ d }

Verification Time for Realistic Orbits

Init (v_1,v_2)	Guard	Transfer Orbit e(c) range	Abstraction Time (s)	MC Time (s)	Real Time (s)
(270,267.5)	330	(0.05849, 0.05853)	811	3.01	(950, 1200)
(250, 246.5)	330	…	811	3.23	(1050, 1300)
(300, 299)	333	(0.06468, 0.06486)	801	3.4	(500, 1250)
(300, 299)	327	(0.06186, 0.06202)	834	3.37	(440, 990

d = 500 km, ε (guard width) = 0.25, partition size 20 x 20

Conclusion

- Contributions
  - Developed **automatic verification technique** of collision avoidance and rendezvous of dual-satellite systems
  - Exposed **limits of unbounded reachability** in periodic systems
  - Produced set of **benchmarks** for nonlinear hybrid systems https://wiki.cites.uiuc.edu/wiki/display/MitraResearch/Satellite+System+Verification

- Looking ahead
  - **Release Tools for hybridization & partitioning**
  - Jeremy Green's Masters Thesis: compositional algorithms much more scalable (forthcoming paper)
  - Verification for satellite formations

Image courtesy NASA

Tables

Table 1. Rendezvous experiments for $d = 500$km, $\epsilon = 0.25$ for guard Λ, and partition size 20×20 degrees. AT is abstraction run time (sec). PT is PHAVer run time (sec). RT is the time interval of rendezvous (sec), with the burn occurring in time at the lower bound of this interval. The underline and overlined parameters e_T or p_T are respectively the min and max of the nondeterministic parameter values.

Initial (ν_1, ν_2)	Guard (ν_1^T, ν_2^T)	Initial Orbit $(e_1, p_1[km], e_I, p_I[km])$	Transfer Orbit $(\underline{e}_T, \overline{e}_T, \underline{p}_T[km], \overline{p}_T[km])$	AT (s)	PT (s)	RT (s)
$(270, 267.5)$	$(330, 330)$	$(0, 6718, 0.05, 7340)$	$(0.05849, 0.05853, 6766, 6769)$	811	3.01	$(950, 1200)$
$(250, 246.5)$	$(330, 330)$	$(0, 6718, 0.05, 7340)$	$(0.05849, 0.05853, 6766, 6769)$	811	3.23	$(1050, 1300)$
$(300, 299)$	$(333, 333)$	$(0.05, 7074, 0.10, 7748)$	$(0.06468, 0.06486, 7114, 7116)$	801	3.4	$(500, 1250)$
$(300, 299)$	$(327, 327)$	$(0, 6718, 0.10, 7748)$	$(0.06186, 0.06202, 6982, 6984)$	834	3.37	$(440, 990)$

Dist(d, o_1, o_2) and Executions