Stability of Distributed Algorithms in the face of Incessant Random Faults

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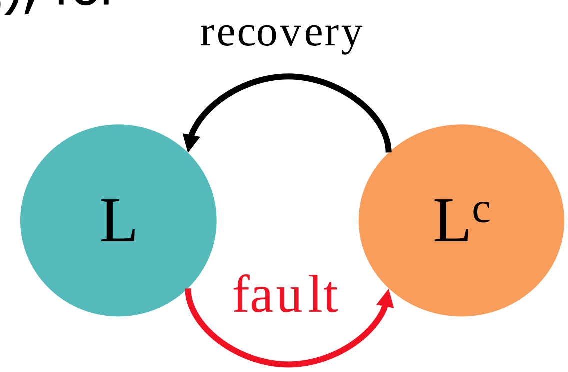
Preview: stabilization time

Given algorithm $A = \langle X, \to \rangle$ and $L \subseteq X$, A is **self-stabilizing** if:

for every $x_0 \in X$, there exists $n(x_0)$, for every $k > n(x_0)$, for every execution $x_0 \to x_1 \to x_2 \ldots x_k$, $x_k \in L$.



Stabilization time $ST(A) = \max_{x \in X} n(x)$

If A starts in L then fault-free executions remain in L

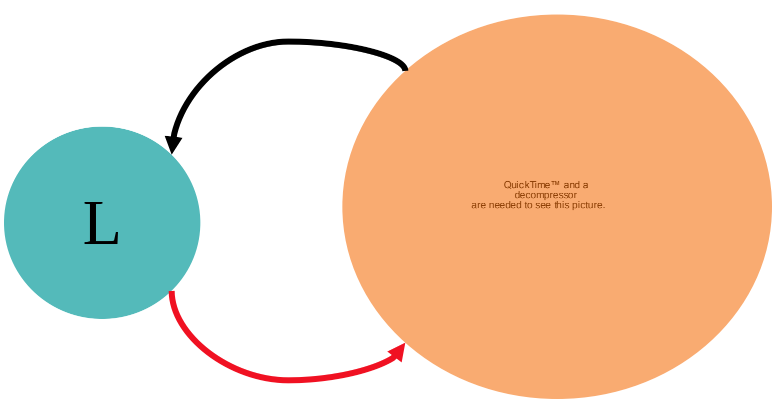
Transient failures may corrupt the state and jump outside L, but if there are **no further failures** then stabilization guarantees that L is reached within ST(A).

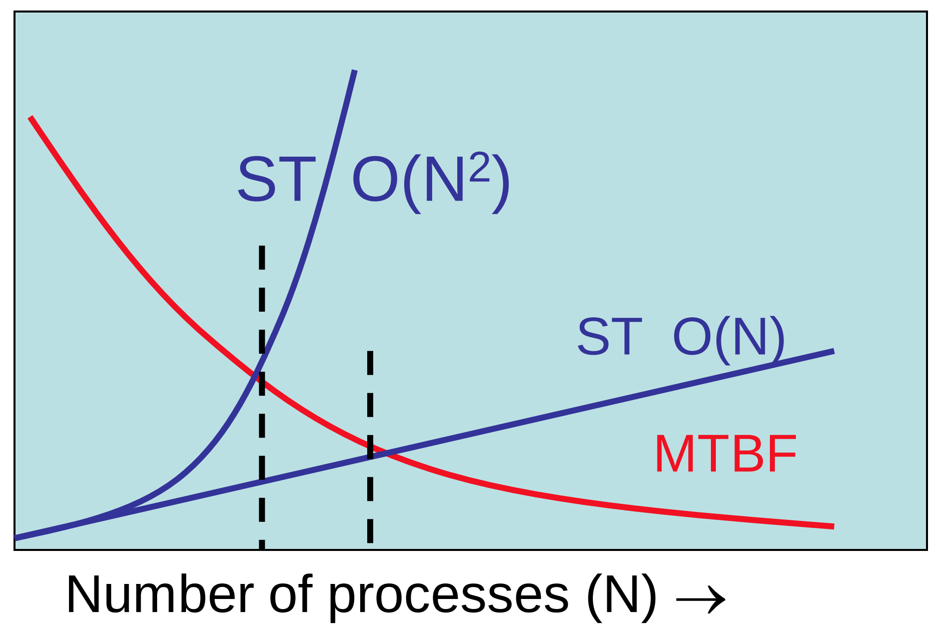
Stabilization in large systems

This talk is concerned with **large distributed** systems

- Typically ST ~ poly(N), where N is # of processes
- MTBF: Mean time between failures
- Large distributed systems

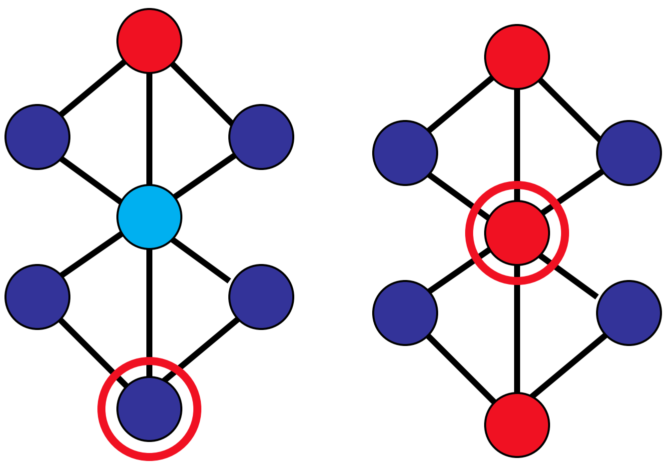
ST >> MTBF



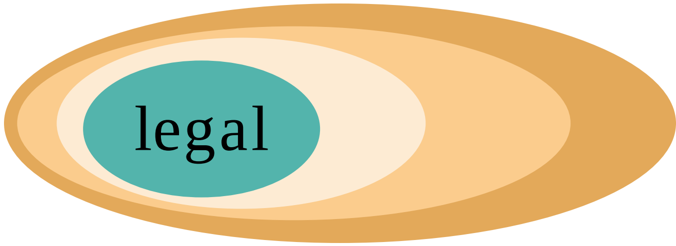


Stability Analysis under Incessant Faults

- All illegal (L^c) states are not equally bad
  - Graph coloring: more conflicts are worse than fewer conflicts
  - Routing: more nodes with optimal routes is better
  - …
- **Define a metric on state space** $Q: X \rightarrow R$
  - $Q(x) = 0$ iff $x \in L$
  - Higher $Q(x)$ means "worse" x



- **Failure rates for process** p_i:
  - e_1 : probability that p_i fails when p_i is supposed to change its state
  - e_2 : probability that p_i fails when p_i is not supposed to change
- Algorithm A defines a stochastic process $A'(e_1,e_2)$
  - Analyze the behavior of A' w.r.t. Q



Outline

- Introduction
- Fault model, example
- Analysis with incessant faults
- An observation and an improvement
- Conclusion

Synchronous Shared Memory Algorithms

- Algorithm $p_i = \langle X_i, U_i, \rightarrow \rangle$
  - X_i: set of **state variables**, U_i: set of **input variables**
  - $\rightarrow_i \subseteq val(U_i) \times val(X_i) \times \wp(val(X_i))$: set of **(probabilistic) transitions**

- Example: Dijkstra's unidirectional token ring
  - N processes 0,1,…,N-1
  - $X_i = \{x_i\}$ where x_i is of type $\{0,1,\ldots,K\}$; $U_i = \{x_{i-1}\}$
  - For $i \neq 0$, if $x_i \neq x_{i-1}$ then $x_i = x_{i-1}$ else $x_i = x_i$
  - For $i=0$, if $x_0 = x_{N-1}$ then $x_0 = x_0 + 1 \% K$ else $x_0 = x_0$

- Complete system $S = p_1 \mid\mid p_2 \mid\mid \ldots \mid\mid p_{N-1}$
  - a state of S: $\mathbf{x}$
  - token($\mathbf{x}$,i) : true iff p_i has token in $\mathbf{x}$
  - hastoken($\mathbf{x}$) = { i | token($\mathbf{x}$,i) }
  - L($\mathbf{x}$) = (|hastoken($\mathbf{x}$)| == 1)

Fault models

- Algorithm $p_i = \langle X_i, U_i, \rightarrow \rangle$
- **Update faults** $e_1>0$, $e_2=0$, for $i \neq 0$
  - if $x_i \neq x_{i-1}$ then
    - $x_i = x_{i-1}$ **with probability** $(1-e_1)$
    - $x_i = k$ **with probability** e_1/K, where $k \in \{0,1,\ldots,K\}$
  - else $x_i = x_i$
- **Sleep-Update faults** $e_1 = e_2 > 0$, for $i \neq 0$
  - if $x_i \neq x_{i-1}$ then
    - $x_i = x_{i-1}$ **with probability** $(1-e_1)$
    - $x_i = k$ **with probability** e_1/K, where $k \in \{0,1,\ldots,K\}$
  - else
    - $x_i = x_i$ **with probability** $(1-e_1)$
    - $x_i = k$ **with probability** e_1/K, where $k \in \{0,1,\ldots,K\}$

Properties of interest

- **Steady State (SS)** distribution π

  $\pi(\{\mathbf{x} \mid |\text{hastokens}(\mathbf{x})| = a\})$
- **Maximum Expected Recovery Time (MERT)**

  $\text{Max}_{\mathbf{x}}$ ERT($\mathbf{x}$), where ERT($\mathbf{x}$) = expected time to reach L from $\mathbf{x}$
- **Maximum Expected Holding Time (MEHT)**

  maximum expected time before a token disappears from a process

Two kinds of analysis

- Model checking (for small N, < 8)
  - Using PRISM model checker
- Asymptotic bounds (for large N)

Steady state probabilities with only update faults

QuickTime™ and a
decompressor
are needed to see this picture.

Maximum Expected Recovery Time

QuickTime™ and a
decompressor
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Update and Sleep-update faults

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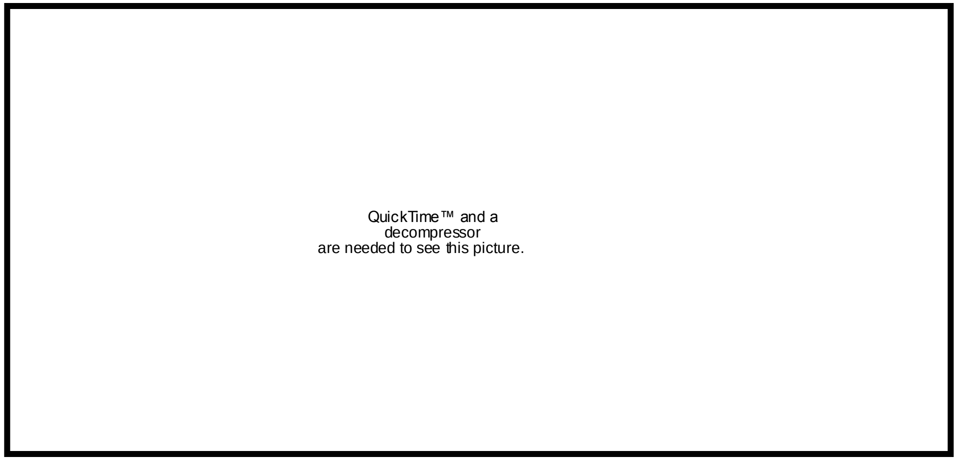
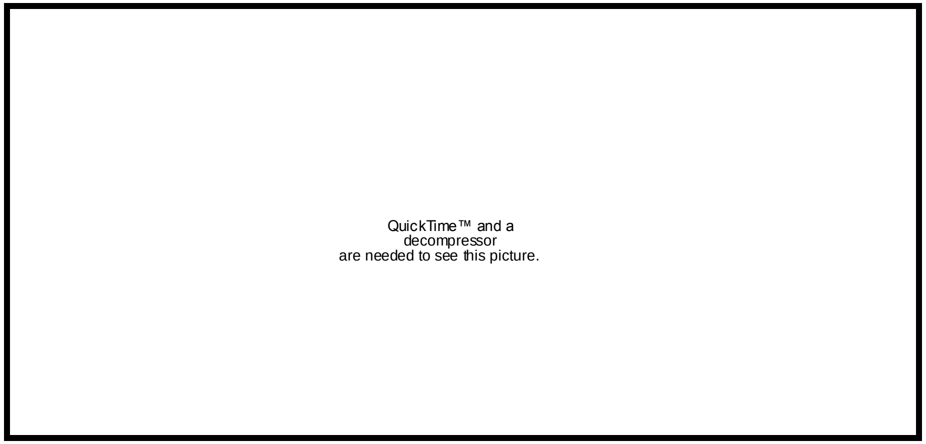
Large N, **update faults** $e_1 = \varepsilon$ $e_2 = 0$



|hastokens()| grows to O(N ε) and then drops to 1



Analysis

- T_t : Number of tokens at round t
- P_t : Process ids for processes with tokens at round t

- When p_i changes value it goes to a correct value with probability $(1-\varepsilon) + \varepsilon/K$ and an incorrect value with probability ε/K (=d)

- Case $T_t = 1$, $L_t = \{i\}$, $i \neq 0$ a a a b b
  - $P(T_{t+1} = 1) = 1- (K-2)d$ a a a a b ; a a a b b
  - $P(T_{t+1} = 2) = (K-2)d$ a a a c b ; a a a d b; ..

- Case $T_t = 1$, $L_t = \{0\}$ a a a a a
  - $P(T_{t+1} = 1) = 1$ a a a a a ; b a a a a; ..

Analysis 2

- Case T_t = **q**, $L_t = \{i,i+1,i+2,..,i+q-1\}$, $0 \notin L_t$
- x_{i-1} x_i x_{i+1} x_{i+2} x_{i+3} … x_{i+q-1} x_{i+q}
  - $P(T_{t+1} = q+1) =$ **d(K-2)** $+ O(d^2)$
  - $P(T_{t+1} = q-1) \le$ **d(2q-1)** $+ O(d^2)$

- Case T_t = **q** in **g** separate groups
  - $P(T_{t+1} = q+1) =$ **εg(K-2)/(K-1)** $+ O(d^2)$
  - $P(T_{t+1} = q-1) \le$ **ε(2q-1)/(K-1)** $+ O(d^2)$

Analysis 3.

- Define stochastic (birth-death) process $Y_0, Y_1, Y_2, \ldots$ as
- $Y_0 = 1$
- $P(Y_{t+1} = Y_t + 1) =$ **ε(K-2) / (K-1)**
- $P(Y_{t+1} = Y_t - 1) =$ **ε(2q-1) / (K-1)**
- $P(Y_{t+1} = Y_t) =$ **1 - ε(K + 2q -3)/(K-1)**

- By Chernoff bound
  - $P(Y_t < T_t) \sim e^{rt}$ for some constant $r < 0$.
  - $Y_N =$ **$N(1-e^{-2\varepsilon})/2$** $+ o(N) \sim \varepsilon N$

- Starting from a single token at p_0 the system generates a token sequence of length **εN** in N steps, with probability exponentially close to 1

Remark on approximate analysis

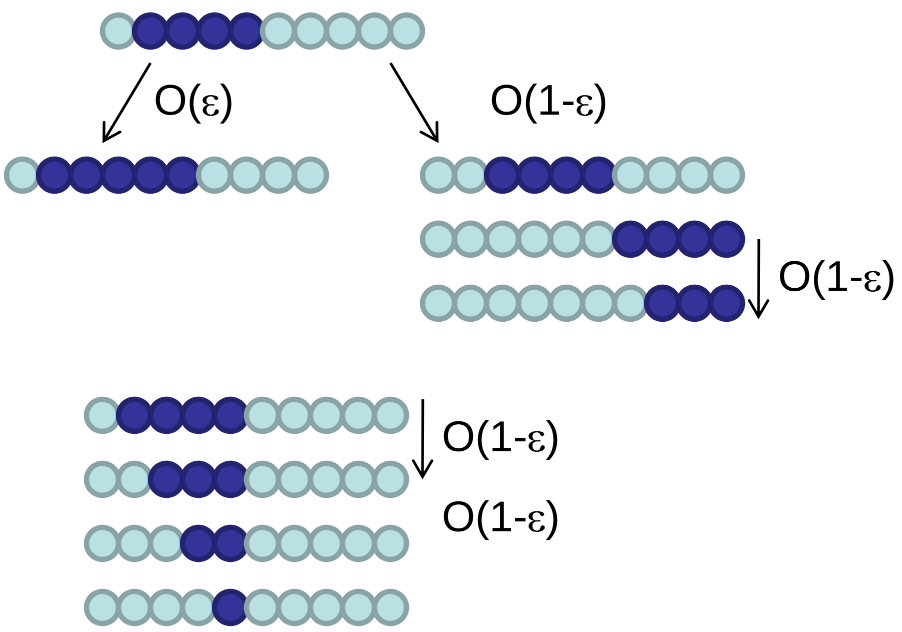
- The preceding analysis was based on approximating the (complex) stochastic process $\{T_t\}$ with a much simpler process $\{Y_t\}$ by conservatively ignoring low probability $O(d^2)$ transitions

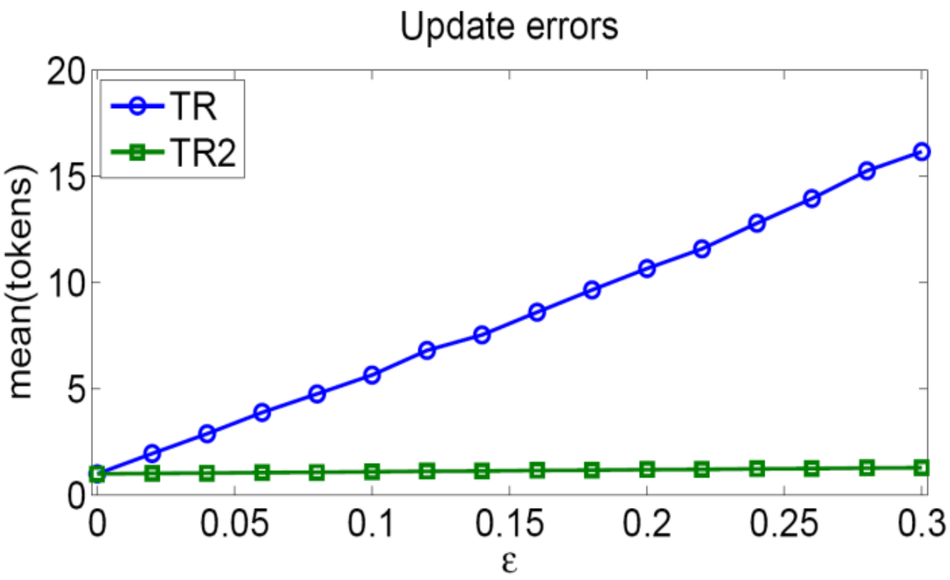
Modified token ring

state of p_j $x_j \in \{0, \ldots, K-1\}$, $K > N$

p_0 :if $x_0 = x_{N-1}$ then $x_0 := x_0 + 1 \mod K$

p_j: if $x_j \neq x_{j-1}$ and $x_{j-1} = x_{j-2}$ then $x_j := x_{j-1}$, for $j > 1$





Comparing steady state distributions for original and modified token ring algorithms

QuickTime™ and a
decompressor
are needed to see this picture.

Model checking results with PRISM for 5 processes

QuickTime™ and a
decompressor
are needed to see this picture.

Graph coloring under U and SU faults

QuickTime™ and a
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Summary

- **Incessant random fault models** and **metrics** for algorithms seems relevant for stabilization in **large systems**
- **Interaction of algorithm and faults** leads to phenomena and modified algorithms
- Analysis using probabilistic model checking and asymptotic analysis

Future directions

- Comparison of existing algorithms specific robustness metrics in the face of incessant faults
- Lower bounds (w.r.t. robustness to incessant faults) and design of new algorithms
- Verification of algorithms under incessant faults (MCs and MDPs)

Related Work

- Random link failures
  - Pelc and Peleg: Feasibility and complexity of broadcasting with random transmission failures: TCS 2007.
  - Kar and Moura: Distributed average consensus in sensor networks with random link failures: IEEE Acoustics, Speech, and Signal processing 2007
- Locally bounded failures
  - Nasterenko and Arora: Local tolerance to unbounded byzantine faults in IEEE SRDS 2002.